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Parallel calculations by automata on direct and back spanning trees of a graph

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Introduction

A task of graph exploration with a goal to uncover a structure of unknown graph by moving along its arcs can be met in many domains. In this paper we regard as possible applications exploration of networks and exploration of Web-application structure. In many cases such an exploration can be considered as being performed by agents working in graph vertices (with possibility to create an agent in a vertex where is no one) and sending each other messages along graph arcs.

Sometimes graph exploration is performed to make possible calculation of some function of values stored in graph vertices. Since different vertices can store equal values, such a function is a function of a multiset of values. We consider situation where calculation is initiated by an external stimulus processes by the automaton located in some root vertex of the graph, and the value calculated is sent by this root automaton to the requester.

Graph processing algorithms and their efficiency highly depend on the known information on the graph, in the worst case in the initial state the graph can be completely unknown and an algorithm should gather information on its structure itself by walking through arcs. Here we suppose that vertex automata are in such states, that contains the following information on the graph structure. Graph has specified direct spanning tree, having the tree root coinciding with the root vertex of the graph and directed from the root, and back spanning tree directed to the root. Arcs not belonging to the direct spanning tree are called chords. Arcs of the direct spanning tree are called direct arcs, of the back one — back arcs. An back arc can be a direct one or a chord. Each vertex automaton stores arc kind (direct, chord, back + direct, back + chord) for each arc outgoing form its vertex. Also vertex automaton stores the number of back arcs incoming to its vertex. Such data can be gathered during previous exploration of the graph described in our paper “Construction of Graph Spanning Tree by Group of Automata”.

Function calculation by vertex automata is performed with *pulse algorithm*. Its idea is first to send request message from the root vertex to all other vertices, then to send response message from each vertex to the root one. It allow to calculate in parallel any function of multiset of values stored in vertex automata (we also sometimes say that they are stored in graph vertices). The worst case working time of the algorithm is $O(D)$, where D is the graph diameter, the maximum length of non-self intersecting path. The details of the algorithm and proofs of all statements can be found in [1].

Aggregate Functions and Aggregate Extension

Pulse algorithm computes so-called aggregate functions, for which function value on a union of multisets can be computed using function values on each of multisets in the union. Here we give definition of an aggregate function and aggregate extension of any given function $f(x)$, which helps to represent it as $h(g(x))$, where g is aggregate. For each f there exists single (up to isomorphism) minimal aggregate extension, which provide a minimum information sufficient to compute f. Aggregate functions theory presented here is a modification of inductive functions theory given in [2].

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Below we consider functions on finite multisets, which elements belong to some base set *X*. The set of all finite multisets of elements of *X* we denote as $X^{\sim}$. Note that union of multisets gives a multiset having all the elements of united ones with multiplicity of any element being a sum of its multiplicities in united multisets.

An aggregate function $g:X^{\sim}\to A$ – is such a function on multisets that
$\exists e:A\times A\to A \ \forall a,b\in X^{\sim} \ g(a\cup b) = e(g(a),g(b))$.

An aggregate extension of function $f:X^{\sim}\to A$ is an aggregate function $g:X^{\sim}\to B$, such that $\exists h : B\to A \ \forall a\in X^{\sim}$ $f(a) = h(g(a))$.

An aggregate extension *g* of *f* is such an aggregate function, that using its values one can compute *f*. Of course, some extensions are not useful, e.g., taking an identity function *g* on $X^{\sim}$, and taking *f* itself as *h*, one doesn't simplify calculations. To avoid such useless solutions, we use a minimal aggregate extension — intuitively, an aggregate function, which give a minimum information sufficient to calculate *f*.

An aggregate extension $g : X^{\sim}\to B$ of $f : X^{\sim}\to A$ is called *minimal*, if $g(X^{\sim}) = B$ and $\forall g` : X^{\sim}\to C$, aggregate extension of *f*, $\exists i : C\to B \ g = ig`$.

Aggregate extension of $f : X^{\sim}\to A$ exists and is unique up to one-to-one mapping.

Pulse Algorithm

Pulse algorithm is intended to calculate a value of *f* on a multiset $x\in X^{\sim}$ of values stored in the graph vertices. We suppose that some value is stored in any vertex with multiplicity 1. Pulse algorithm uses the graph structure information stored in vertex automata as it is described in the Introduction.

Pulse algorithm uses two message kinds: **Request** and **Response**. At first the root vertex automaton gets **Request** from some external source, this message provides three functions: *h*, *e*, and *g*. This message is then transmitted along direct arcs to all other vertices, and all vertex automata store *e* and *g*. Each vertex automaton then calculates the value of *g* on values stored in the vertices of subtree with the root coinciding with this vertex of back spanning tree, and sends the value obtained as a parameter of **Response** message along back arcs. The root vertex automaton calculates *g(x)*, and then sends **Response** to the request initiator with the value *f(a)=h(g(x))*.

Pulse algorithm worst case working time is ***O****(D)*.

References

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